Optimal advertising media allocation under fuzzy environment for a multi-product segmented market

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Abstract

Advertising & promotion are an integral part of our social & economic system. An advertising decision is inclined by the choice of media, media effectiveness, media budget etc. especially if the advertising is required to be done in a segmented market. However, many of the decision making problems in the real world take place in an environment in which the goals, the constraints, and the consequences of possible actions are not known precisely. These problems are more inherent in advertising as budget on advertising demands huge investments. Planning for suitable media options and then judicious allocation of the available advertising budget amongst them involves critical thinking. Fortunately, fuzzy logic helps to provide solution to situations which involves decision making under fuzzy environment. This paper presents a similar problem which involves media planning and allocation problem for advertising consumer durable products in a segmented market. The potential market consists of industrial users as well as domestic users which uses the common range of products. The inherent costs of media advertising are fuzzy in nature. In order to resolve the strategic decision making related to advertisements of these products in a segmented potential adopter market under fuzzy environment, the problem is formulated as a fuzzy multi-objective optimization problem. The objective is to determine the optimal number of advertisements to be allocated to different media so as to maximize the customer increase rate and hence advertising effectiveness subject to fuzzy advertising cost budget constraint. The funds available for investment in different media are uncertain.

Fuzzy goal programming approach has been used to achieve a compromised solution. The numerical illustrates a firm manufacturing consumer durables which uses three different media such as print media (which includes various newspapers as well as
magazines), website media and T.V media as medium of advertising. The satisfying solution is identified and analyzed, and sensitivity analyses are performed for model flexibility by providing different weights combinations to different products in each segment so as to meet the aspiration levels of management for each product.

**Keywords:** Advertising & media planning, media allocation, segmented market, fuzzy goal programming

1. Introduction

Communicating effectively and efficiently with the consumers is critical to the success of both private and public sector companies. Businesses plan their marketing initiatives by examining the marketing mix, a blend of activities such as designing the product and its package, pricing the product, distributing the product so that it is accessible to customers and promoting & communicating about the product. The ways of promoting a product determines whether a firm successfully manifests the right messages in the minds of their target audience or not. Among the key elements of promotional mix which constitutes advertising, personnel selling, publicity and sales promotion, advertising is the most important. Advertising can be done through various media options available such as print media or electronic media. Planning for a suitable media is of prime concern for any marketing manager as it influences the use of time and space to achieve advertising objectives such as placing an advertising message at right place and at right time before a target audience. This involves selecting an appropriate advertising media and development and allocation of the suitable advertising budget to these media. The media are chosen specifically for each of the targeted segment so as to capture maximum from it. These segments are the broad classes of buyers who have the same needs and will respond similarly to marketing actions. For example, if we are targeting rural areas, then radio or television could be an appropriate media whereas for metropolitan cities, newspapers, magazines, television as well as internet could be used. On the other side, types of media, number of products to be advertised, expected customer increase rate of the company’s major products, the frequency of advertisements, etc. are various factors that determine the allocation of the firms’ advertising budget. Since the amount of available budget is limited and fixed, it is desired to spend the available budget judiciously so as to obtain maximum exposure for all the products that needs to be advertised in different market segments.

Many of the decision making problems in the real world take place in an environment in which the goals, the constraints, and the consequences of possible actions are not known precisely (Bellman and Zadeh, 1970). Also there exists vagueness in the audience responses to real life circumstances as there exists many phenomena in human life which can hardly be defined with certainty. The vagueness under discussion is not due to randomness. It occurs because human capability to understand and analyze imprecise events is troublesome. Such situations make a real decision problem very complicated and thus often seems to be little suited to mathematical modeling because there is no crisp definition. The optimal solution of the problem so obtained, is not actually representative of the complete & exact information. Implementation of such solution
may result in huge losses due to vague definition of the proposed model. Fuzzy set theory (Zadeh (1965), Zimmermann (1991), Lee (2005)) builds a model to represents a subjective computation of possible effect of the given values on the problem and permits the incorporation of vagueness in the conventional set theory that can be used to deal with uncertainty quantitatively.

Most of the quantitative modeling tools that are available to solve media planning problems are classified as simulation, heuristic, or multi-criteria decision-making models. These models use goal programming, linear programming, analytic hierarchy process techniques etc. to obtain satisfactory solution of the original problem. Few studies have explored the conflicting media planning issues in terms of customer relationships, advertising effects, and resource allocation. All these problems have considered all market segments alike. In reality each market segment is unique and hence requires separate attention. In this paper we develop media selection model for multiple products in market with multiple segments under fuzzy environment. A case of media planning and allocation is considered for a firm who wishes to advertise its range of consumer durable products amongst multiple segments of potential adopter population. The segments are distinguished between students and professionals. Students segment comprises of teenagers and the campus going crowd whereas professionals are the employees of multinational companies or small/large business professionals. The objective is to allot optimal number of advertisements in selected media categories given the total budget, so as to maximize the expected customer increase rate for each product. The model has been formulated in such a way that the advertisement should reach to those who are potential for the product instead of going to those section who are not potential for the product. Since the data pertaining to the per unit advertisement cost as well as the total advertising cost budget is fuzzy or uncertain and the problem involves maximizing the expected number of customers for each product in all segments simultaneously while adhering to its cost budget and technical constraints, the problem is formulated as a multi-objective fuzzy mathematical programming problem. In order to obtain a compromise feasible solution to the formulated problem, fuzzy goal programming technique as suggested by Mohamed (1995) have been used.

Literature review of media selection and allocation and fuzzy optimization techniques is discussed in Section 2. Section 3 discusses model development and formulation. Solution methodology of the developed model is provided in Section 3.2. Case study has been discussed in Section 4 to illustrate the solution methodology. Section 5 explains the application of fuzzy goal programming approach to the problem. Numerical results are presented in section 6. Concluding remarks are made in section 7.

2. Literature review

Research in the field of media selection and allocation began with work of Bass & Lonsdale (1966) who explored the use of linear programming in media selection. In particular they examined the influence of various methods of weighing the audience data to be used in linear program. Charnes et al. (1968) introduced a Goal Programming
model for media selection to address problems associated with the critical advertising measurement of frequency and reach. Such similar problem was considered by Lee (1972) as well. The media selection models were also addressed using MCDM modeling techniques. The study on improvements in media selection methods was based on generalized GP research by Kendall (1975). De Kluyver (1978) proposed the more realistic use of hard and soft constraints for linear programming models used in media selection. Keown and Duncan (1979) developed an integer GP model to solve media selection problems and improve upon suboptimal results produced by linear programming and non-integer GP models. The result provided examples of integer GP formulations that overcome most of the limitations found in earlier linear programming models. Hoffman et al. (1996) identified an approach to modeling the advertising planning process. They determined a test city’s critical market characteristics that fit most advantageously with the corporation’s marketing strategy. Lee and Kwak (1999) have developed an information resource planning using an AHP based goal programming model. An optimal advertising pulsation policy through dynamic programming approach was derived by Mesak and Zhang (2001). An approach to planning an advertising campaign of goods and services was discussed by Belenky (2001, 2002) in conformity to single as well as multiple products which are advertised by a firm within a certain period of time. Under certain natural assumptions on the behavior of a potential consumer, problems of planning the campaign was formulated as nonlinear programming ones with linear constraints of a balance kind that can be solved by well-developed nonlinear programming methods. Mihiotis and Tsakiris (2004) reviewed the recent study related to advertising planning. The study discussed the best possible combination of placements of a commercial (channel, time, and frequency) with the goal of the highest rating subject to constrained advertising budgets. Kwak et al. (2005) have presented a case study that considers two options: industrial and consumer products. A mixed integer goal programming model was used to resolve the strategic decision making about dual market high technology products and to facilitate their advertising media selection process. A linear programming approach for determining optimal advertising policy by Ching et al. (2006) was based on the seminal works done by Mesak and Zhang (2001). He proposes an advertising model which can capture the advertising wear out phenomenon. The aim was to derive an optimal pulsation advertising strategy. The optimization problem was formulated as linear programming problem. A chance constraints goal programming model for the advertising planning problem by U.K. Bhattacharya (2009) presents a model which has been designed to decide the number of advertisement in different advertising media and the optimal allocation of the budget assigned to the different media. But these models ignore the practical aspect of segmentation.

**Research in the field of fuzzy optimization**

Fuzzy optimization is a flexible approach that permits more adequate solutions of real problems in the presence of vague information, providing the well defined mechanisms to quantify the uncertainties directly. The fuzzy set concept and fuzzy optimization techniques can be used efficiently in such situations to defuzzify the fuzzy parameters, constraints and objectives (Heilpern (1992) and formulating an equivalent crisp problem which can further be solved using the mathematical programming techniques. The idea of fuzzy programming was first given by Bellman and Zadeh (1970) and then developed
by Tanaka (1974), Zimmermann (1976, 1978, and 1985). A number of researchers thereafter have contributed to the development of fuzzy optimization technique (Zimmermann, (1991); Mohamed, (1997); Bector and Chandra, (2005) and Lee, (2005) etc). Recently Saneifard (2011) has introduced a new approach for defuzzification of trapezoidal fuzzy numbers based on distance method. This method considers the centroid point of a fuzzy number but also the minimum crisp value of fuzzy number. Today, similar to the developments in crisp optimization, different kinds of mathematical models have been proposed and many practical applications have been implemented by using the fuzzy set theory. Gladish et al. (2010) develops a crisp multi-objective programming model from the fuzzy basic data. When a feasible solution to the problem exists, single and multiple objective fuzzy optimization procedure are used to solve the problem otherwise fuzzy goal optimization is used.

**Contribution of present research**

Present literature lacks the decision making under fuzzy environment for a media selection and allocation problems. Also the concept of segmentation was somehow ignored too. Recently Jha et al. (2011) have formulated an optimal media selection model for a multi-product segmented market but the problem has not been considered under fuzzy implications of audience impact. Also the concept of management aspirations from the product market i.e the expected customer increase rate corresponding to firm’s products was also lacking. It is tried to remove some of these deficiencies. A model has been developed which deals with determining the optimal number of advertisements in each media and allocate the advertising media budget to selected media categories given the total budget, so as to maximize the customer increase rate for each product. All this has been incorporated considering that the audience responses are not exact or certain but varies i.e. under fuzzy environment.

**3. Model Development**

The following section develops and formulates a multi-objective media planning model for multiple products under fuzzy environment in a segmented market. The objective is to maximize the expected number of customers while adhering to advertising budget. The following parameters, indices and decision variables have been used as a part of the model development.

**Notations**

\( i=1,2,...,M \) segments  
\( j=1,2,...,N \) medium of advertising  
\( p=1,2,...,P \) products  
\( k_{j} \): media options of \( j^{th} \) medium  
\( l_{j} \): insertion /slot of \( j^{th} \) medium  
\( Z_{pi} \): expected increase in number of customers of \( p^{th} \) product in \( i^{th} \) segment  
\( r_{pikj} \): expected customer increase rate for \( p^{th} \) product in \( i^{th} \) segment, \( j^{th} \) media, \( k_{j}^{th} \) media option & \( l_{j}^{th} \) slot / insertion
\( x_{pijk,l} \) : decision variable corresponding number of advertisements for \( p \)th product in \( i \)th segment, \( j \)th media, \( k \)th media option & \( l \)th slot / insertion

\( \tilde{c}_{pijk,l} \) : triangular fuzzy advertising cost of inserting one advertisement for \( p \)th product in \( i \)th segment, \( j \)th media, \( k \)th media option & \( l \)th slot / insertion

\( c_{pijk,l} \) : defuzzified unit cost of advertisement for \( p \)th product in \( i \)th segment, \( j \)th media, \( k \)th media option & \( l \)th slot / insertion

\( t_{pijk,l} \) : minimum number of advertisements for \( p \)th product in \( i \)th segment, \( j \)th media, \( k \)th media option & \( l \)th slot / insertion

\( u_{pijk,l} \) : maximum number of advertisements for \( p \)th product in \( i \)th segment, \( j \)th media, \( k \)th media option & \( l \)th slot / insertion

\( A_i \) : fuzzy advertising budget for \( i \)th segment

\( A_i^0 \) : defuzzified advertising budget available for all the products in \( i \)th segment

\( A_i(\chi) \) : defuzzified total desired advertising budget all the products in \( i \)th segment

### 3.1 Model formulation

The model formulation must provide a satisfying mix of advertising media expenditures that meet the customer increase rate objective for each type of product, while adhering to the limitations of media resource availability in a segmented market. Expected customer increase rate is the rate at which the customers are expected to increase once they are aware of the product and hence reflects the last stage of product adoption cycle. The problem of selecting appropriate media that will maximize the total advertising reach and also the expected customer increase rate for different products that can be marketed amongst different segments \((i=1,2,...,m)\) under different media options and fuzzy cost–budget constraint can be written as a multi-objective programming problem involving different products under different market segments.

Maximize

\[
\begin{align*}
Z_{p1} &= \sum_{j=1}^{N} \sum_{k=1}^{K} \sum_{l=1}^{L} x_{p1jk,l} c_{p1jk,l}, \quad p = 1,2,...P \\
Z_{p2} &= \sum_{j=1}^{N} \sum_{k=1}^{K} \sum_{l=1}^{L} x_{p2jk,l} c_{p2jk,l}, \quad p = 1,2,...P \\
&\vdots \\
Z_{pn} &= \sum_{j=1}^{N} \sum_{k=1}^{K} \sum_{l=1}^{L} x_{pnjk,l} c_{pnjk,l}, \quad p = 1,2,...P
\end{align*}
\]

(P1)
Subject to

\[ A_i(X) = \sum_{p=1}^{N} \sum_{j=1}^{L} \sum_{k=1}^{N} c_{pjik} x_{pjik} \leq A_i^0 \quad :i = 1, 2, \ldots, M \]

\[ Z_{pi}^0(X) = \sum_{j=1}^{L} \sum_{k=1}^{N} r_{pjik} x_{pjik} \geq Z_{pi}^0 \quad \forall p = 1, 2, \ldots, P ; i = 1, 2, \ldots, M \]

Since the cost budget constraint as well as cost parameters are fuzzy in nature, we need to use a proper solution methodology to obtain a feasible solution to the above mentioned multiple objective programming problem.

3.2 The Solution methodology

Following are the sequential steps to solve the fuzzy mathematical programming problems.

1. Compute the crisp equivalent of the fuzzy parameters using a defuzzification function. Same defuzzification function is to be used for each of the parameters. Here we use the Heilpern’s defuzzification function

\[ F_{\frac{1}{2}}(A) = \frac{a_1^2 + 2a_2^2 + a_3^2}{4} \]

where \( a_1, a_2, a_3 \) are the triangular fuzzy numbers (Heilpern (1992)).

2. An imprecise aspiration levels has been assigned to the objective by incorporating the objective function as a fuzzy constraint with a restriction (aspiration) level. The inequalities are defined softly if the requirement (resource) constants are defined imprecisely. The above problem (P1) can be rewritten as

Find \( X \)

Subject to

\[ A_i(X) \leq A_i^0 \quad \forall i = 1, 2, \ldots, M \]

\[ Z_{pi}^0(X) = \sum_{j=1}^{L} \sum_{k=1}^{N} r_{pjik} x_{pjik} \geq Z_{pi}^0 \quad \forall p = 1, 2, \ldots, P ; i = 1, 2, \ldots, M \]

\( X \in S \quad (P2) \)

3. Define appropriate membership functions for each fuzzy inequalities as well as constraint corresponding to the objective function. The membership function for the fuzzy less than or equal to and greater than or equal to type are given as

\[ \mu_A(X) = \begin{cases} 1 & : A_i(X) \leq A_i^0 \\ \frac{1 - (A_i^0 - A_i(X))}{A_i^0 - A_i^*} & : A_i^0 < A_i(X) < A_i^* \\ 0 & : A_i(X) > A_i^* \end{cases} \]

\[ \mu_{Z_{pi}^0}(X) = \begin{cases} 1 & : Z_{pi}^0(X) = Z_{pi}^0 \\ \frac{Z_{pi}(X) - Z_{pi}^0}{Z_{pi}^0 - Z_{pi}^*} & : Z_{pi}^0 < Z_{pi}^0(X) < Z_{pi}^* \end{cases} \]

respectively, where \( A_i^0 \) and \( Z_{pi}^0 \) are the restriction and aspiration levels respectively of the cost budget & customer increase rate and \( A_i^* \) and \( Z_{pi}^* \) are the tolerance levels corresponding to the \( i^{th} \) segment.
4. Employ extension principle (Bellman & Zadeh (1970)) to identify the fuzzy decision, which results in a crisp mathematical programming problem given by

Maximize $\alpha$

Subject to

$\mu_{A_i}(X) \geq \alpha$, $i = 1, 2, ..., M$

$\mu_{Z_{pi}}(X) \geq \alpha \ \forall p = 1, 2, ..., P; i = 1, 2, ..., M$

$X \in S; \ \alpha \in [0, 1]$

and can be solved by the standard crisp mathematical programming algorithms.

5. While solving the problem following steps 1-4, objective of the problem is also treated as a constraint. Each constraint is considered to be an objective for the decision maker and the problem can be looked as a fuzzy multiple objective mathematical programming problem. Further each objective can have different level of importance and can be assigned weight to measure the relative importance. The resulting problem can be solved by the weighted $\min\max$ approach. The crisp formulation of the weighted problem is given as (P4)

Maximize $\alpha$

Subject to

$\mu_{A_i}(X) \geq \alpha$, $i = 1, 2, ..., M$

$\mu_{Z_{pi}}(X) \geq w_{pi}\alpha \ \forall p = 1, 2, ..., P; i = 1, 2, ..., M$

$X \in S; \ \alpha \in [0, 1]$

$w_{pi} \geq 0 \ , p = 1, 2, ..., P; i = 1, 2, ..., M$

$\sum_{p=1}^{P} w_{pi} = 1; \forall i = 1, 2, ..., M$

where $m$ is the number of constraints in (P4) and $\alpha$ represents the degree up to, which the aspiration of the decision maker is met. If the constraints are fuzzy as well as crisp, then in the equivalent (crisp) linear programming problem, the original crisp constraints will not have any change as for them the tolerances are zero. The problem (P4) can be solved using standard mathematical programming approach.

6. On substituting the values for $\mu_{A_i}(X)$ & $\mu_{Z_{pi}}(X)$ the problem becomes

Maximize $\alpha$

Subject

$A_i(X) \leq A_i^0 + (1 - \alpha)(A_i^* - A_i^0)$

$Z_{pi}(X) \geq Z_{pi}^0 - (1 - w_{pi}\alpha)(Z_{pi}^0 - Z_{pi}^*)$

$w_{pi} \geq 0$

$\sum_{p=1}^{P} w_{pi} = 1 \ \forall i = 1, 2, ..., M$

$\mu_{Z_{pi}}(X) \geq w_{pi}\alpha \ \forall p = 1, 2, ..., P; i = 1, 2, ..., M$

$X \in S; \ \alpha \in [0, 1]$

7. If a feasible solution is not obtainable for the problem (P4) or (P5) then we can use fuzzy goal programming approach to obtain a compromised solution. In the literature there are many approaches to solve the problem (FGP). Zimmerman’s
approach (1991) and crisp goal programming approach given by Mohamed (1997) are the most prominent ones to find a compromise solution to the programming problem. We have used Mohamed (1997) minsum GP approach. The method is discussed in detail in the numerical illustration.

4. Case problem

Nowadays consumer durables such as laptops, notebooks etc. are in demand. Looking at the interest of the people many companies are launching different versions or models in this category. Take for example the case of Dell. It has launched a whole array of laptops, notebooks & scanners / printers. For example, in their everyday essentials range, it has launched the Inspiron 14R, Inspiron 15R, Inspiron R. Then there are laptops with wireless displays (WiDi). Similarly in the mini notebook range it offered Mini1012, Mini1018 version. Other than this there is Vostro range which offered laptops for small business professionals. One such similar company has been discussed as a case in this paper. This company manufactures various models of laptops, mini notebooks, printers / scanners and its related accessories for students as well as professionals. The name of the company being studied is not released for confidentiality. Three products namely Laptops, notebooks & Printers/scanners have been considered which are being advertised through different business publications, magazines, newspapers, internet media, and spot television. Also there is a restriction on the minimum & maximum number of advertisements that can be placed in a media. The main aim is to advertise in different media so as to maximize the expected number of customers (measured in terms of customer increase rate) to the target segments with in its allowable budgets assigned for the different media without violating the maximum and minimum number of advertisements for various media. During research, experts were asked to give their views regarding the relative importance of various constraint factors. The data pertaining to cost coefficients is a triangular fuzzy data. Problem is formulated as multi-objective programming problem.

Customer increase rate for different products in different segments, advertising presence in publications for different segments for laptops, notebooks & printers/scanners in different slots is provided in Tables 4, 5, 6, 7 of the Appendix respectively. On using the mentioned data & the defuzzified values of the cost budget coefficients for all products, a multi-objective programming problem combining all the objectives that will maximize the customer reach for professionals segment subject to the advertising cost budget constraint for three different products (p=1,2,3) , two market segments & four different media options can be formulated as follows:

**Segment: Professionals**

Maximize

\[ Z_{p1} = \sum_{j=1}^{4} \sum_{k_1=1,2; k_2=1,2} \sum_{l_3=1}^{3} r_{p1,jk,lj} x_{p1,jk,lj} \quad \forall \ p = 1, 2, 3 \]

subject to

\[ \sum_{p=1}^{3} \sum_{j=1}^{4} \sum_{k_1=1,2; k_2=1,2} \sum_{l_3=1}^{3} c_{p1,jk,lj} x_{p1,jk,lj} \leq A_i \]

\[ Z_{11} \geq Z_{11}^0; Z_{21} \geq Z_{21}^0; Z_{31} \geq Z_{31}^0; \]

\[ X_i \in S_i; \quad X_i \geq 0 \& \text{integers} \]
The triangular fuzzy values for the advertising cost budget for this segment is (Rs 100,00,000 ; Rs 125,00,000 ; Rs 150,00,000). This gives the defuzzified value of the budget for this segment as Rs 125,00,000 (using Heilpern’s defuzzifier). The aspirations level of the management lies in obtaining the expected number of customers if around 40-45% of the total advertising budget (of the professional segment) is allocated to each product category. In other words, the optimal values for $Z_p$ for $p=1,2,3$ is obtained when approximately Rs 50,00,000 is allocated to each of the products as $[Z_p^1 = 45018, Z_p^2 = 69385, Z_p^3 = 30000]$. Incorporating these aspiration levels, the problem is further solved.

Problems (P6) can be written as the multi-objective programming problem (P7) as

\[
\text{Find} \quad X_1 = [x_{pikj}], \quad p = 1, 2, 3; \quad i = 1; \quad j = 1, 2, 3, 4; \quad k_1 = 1, 2, 3, 4; \quad k_2 = 1, 2, 3, 4; \quad k_3 = 1, 2; \quad k_4 = 1, 2; \quad l_j = 1, 2
\]

Subject to

\[
\sum_{p=1}^{3} \sum_{j=1}^{4} \sum_{k_1=1}^{2} \sum_{k_2=1}^{4} \sum_{k_3=1}^{2} \sum_{k_4=1}^{2} \sum_{l_j=1}^{2} c_{pikj} x_{pikj} \leq 12500000 \quad \forall p = 1, 2, 3 \tag{P7}
\]

\[
Z_{11} \geq 45018; \quad Z_{21} \geq 69385; \quad Z_{31} \geq 30000; \\
X_1 \in S_1; \quad X_1 \geq 0 \& \text{integers}
\]

Similarly for the students segment, multi-objective programming problem maximizing the expected number of customers subject to the advertising cost budget constraint for three different products ($p=1,2,3$), two market segments & four different media options can be formulated as follows:

**Segment: Students**

Maximize

\[
Z_p^2 = \sum_{p=1}^{3} \sum_{k_1=1}^{2} \sum_{k_2=1}^{2} \sum_{k_3=1}^{3} \sum_{k_4=1}^{2} \sum_{l_j=1}^{2} c_{pikj} x_{pikj} \quad \forall p = 1, 2, 3
\]

subject to

\[
\sum_{p=1}^{3} \sum_{k_1=1}^{2} \sum_{k_2=1}^{2} \sum_{k_3=1}^{3} \sum_{k_4=1}^{2} \sum_{l_j=2}^{2} c_{pikj} x_{pikj} \leq A_i \tag{P8}
\]

\[
Z_{12} \geq Z_{22}; \quad Z_{22} \geq Z_{32}; \quad Z_{32} \geq Z_{42}; \\
X_2 \in S_2; \quad X_2 \geq 0 \& \text{integers}
\]

Where

\[
X_2 = [x_{pikj}], \quad p = 1, 2, 3; \quad i = 2; \quad j = 1, 2, 3, 4; \quad k_1 = 1, 2, 3, 4; \quad k_2 = 1, 2, 3, 4; \quad k_3 = 1, 2; \quad k_4 = 1, 2; \quad l_j = 1, 2
\]

\[
S_2 = [x_{pikj}, \geq u_{pikj}], \quad p = 1, 2, 3; \quad i = 2; \quad j = 1, 2, 3, 4; \quad k_1 = 1, 2, 3, 4; \quad k_2 = 1, 2, 3, 4; \quad k_3 = 1, 2; \quad k_4 = 1, 2; \quad l_j = 1, 2
\]
The triangular fuzzy values of the advertising cost budget for this segment is (Rs130,00,000; Rs115,00,000; Rs 80,00,000). This gives the defuzzified value of the budget for this segment as Rs110,00,000 (using Heilpern’s defuzzifier). Allocating around 40-45% of this budget individually to each product provides the expected customer increase rate that the management wants to achieve for each product in this segment. Therefore the optimal values for \( Z_{p2} \) for \( p = 1, 2, 3 \) obtained when approximately Rs 50,00,000 is allocated to each of the products as \( Z_1 = 44656, Z_2 = 53454, Z_3 = 34624 \). With the required aspiration levels, the problem is further solved.

Problems (P8) is rewritten as the multi-objective programming problem (P9) as

**Find**

\[ X_2 = \{ x_{pjk,l_j} \}, p = 1, 2, 3; i = 2; j = 1, 2, 3, 4; k_1 = 1, 2, 3, 4; k_2 = 1, 2, 3, 4; k_3 = 1, 2; k_4 = 1, 2; l_j = 1, 2 \]

**Subject to**

\[ \sum_{p=1}^{3} \sum_{j=1}^{4} \sum_{k_1=1, 2, 4} \sum_{k_2=1, 2} \sum_{k_3=1, 2} c_{pjk,l_j} x_{pjk,l_j} \leq 11000000 \quad \forall p = 1, 2, 3 \quad \text{(P9)} \]

\[ Z_{12} \geq 44656; Z_{22} \geq 53454; Z_{32} \geq 34624; \]

\[ X_2 \in S_2; X_2 \geq 0 \& \text{integers} \]

Since inequalities corresponding to advertising cost budget as well as reach objectives are still fuzzy, membership functions for each of the fuzzy constraint needs to be defined. For the two segments i.e professionals and students segment the membership functions are given as follows:

**Segment: Professionals**

\[ \mu_{A_i}(X) = \begin{cases} 
1 & \text{if } 12500000 - \sum_{p=1}^{3} \sum_{j=1}^{4} \sum_{k_1=1, 2, 4} \sum_{k_2=1, 2} \sum_{k_3=1, 2} c_{pjk,l_j} x_{pjk,l_j} \leq A_i \\
0 & \text{if } 12500000 - \sum_{p=1}^{3} \sum_{j=1}^{4} \sum_{k_1=1, 2, 4} \sum_{k_2=1, 2} \sum_{k_3=1, 2} c_{pjk,l_j} x_{pjk,l_j} > A_i \\
\end{cases} \]

The customer increase rate for each of the three products is given by

\[ \mu_{Z_{11}}(X) = \frac{1}{4} \left( \sum_{j=1}^{4} \sum_{k_1=1, 2} \sum_{k_3=1, 2} x_{1jk,l_j} n_{1jk,l_j} - 40000 \right) \]

\[ \begin{cases} 
1 & \text{if } 40000 \leq Z_{11}(X) < 45000 \\
0 & \text{if } Z_{11}(X) < 40000 \\
\end{cases} \]
\[
\mu Z_{21}(X) = \begin{cases} 
\sum_{j=1}^{4} \sum_{k_1=1,2}^{4} \sum_{k_2=1,2}^{4} r_{21j}k_jl_jx_{21j}k_jl_j - 65000 \\
0 
\end{cases} 
\]

\[
\mu Z_{31}(X) = \begin{cases} 
\sum_{j=1}^{4} \sum_{k_1=1,2}^{4} \sum_{k_2=1,2}^{4} r_{31j}k_jl_jx_{31j}k_jl_j - 25000 \\
0 
\end{cases} 
\]

Segment: Students

\[
\mu A_2(X) = \begin{cases} 
\sum_{p=1}^{4} \sum_{j=1}^{4} \sum_{k_1=1,2}^{4} \sum_{k_2=1,2}^{4} c_{p2j}k_jl_jx_{p2j}k_jl_j \leq \delta_1 \\
0 
\end{cases} 
\]

The customer increase rate for each of the three products is given by

\[
\mu Z_{12}(X) = \begin{cases} 
\sum_{j=1}^{4} \sum_{k_1=1,2}^{4} \sum_{k_2=1,2}^{4} r_{12j}k_jl_jx_{12j}k_jl_j - 40000 \\
0 
\end{cases} 
\]

\[
\mu Z_{22}(X) = \begin{cases} 
\sum_{j=1}^{4} \sum_{k_1=1,2}^{4} \sum_{k_2=1,2}^{4} r_{22j}k_jl_jx_{22j}k_jl_j - 50000 \\
0 
\end{cases} 
\]

\[
\mu Z_{32}(X) = \begin{cases} 
\sum_{j=1}^{4} \sum_{k_1=1,2}^{4} \sum_{k_2=1,2}^{4} r_{32j}k_jl_jx_{32j}k_jl_j - 30000 \\
0 
\end{cases} 
\]
The equivalent crisp optimization problems based on extension principle corresponding to problem (P7) & (P9) for \( i=1,2 \) segments considering different weights for the cost and advertising reach objectives, is given as problem (P10a) & (P10b) respectively.

Segment: Professionals

\[
\begin{align*}
\text{Maximize } & \quad a \\
\text{Subject to } & \\
\mu_{A_1}(X) & \geq \alpha \\
\mu_{Z_{p1}}(X) & \geq w_{pi} \alpha \quad \forall \ p = 1, 2, 3 \\
\sum_{p=1}^{3} w_{pi} &= 1 \\
0 & \leq \alpha \leq 1 \\
X_i \in S_1; & \ X_i \geq 0 \text{ & integers} \\
\end{align*}
\]

Segment: Students

\[
\begin{align*}
\text{Maximize } & \quad a \\
\text{Subject to } & \\
\mu_{A_2}(X) & \geq \alpha \\
\mu_{Z_{p2}}(X) & \geq w_{pi} \alpha \quad \forall \ p = 1, 2, 3 \\
\sum_{p=1}^{3} w_{pi} &= 1 \\
0 & \leq \alpha \leq 1 \\
X_i \in S_2; & \ X_i \geq 0 \text{ & integers} \quad P(10a) \\
\end{align*}
\]

The problem is a crisp linear programming problem and can be solved using standard mathematical programming methods.

5. Fuzzy Goal Programming Approach

On solving the problem, we found that the problem (P10a) & (P10b) are not feasible, hence the management goal can’t be achieved for a feasible value of \( \alpha \in [0, 1] \). Now we use fuzzy goal programming technique to obtain a compromised solution. The approach is based on the goal programming technique for solving crisp goal programming problem [Mohamed (1997)]. The maximum value of any membership function can be 1; maximization of \( \alpha \in [0, 1] \) is equivalent to making it as close to 1 as best as possible. This can be achieved by minimizing the negative deviational variables of goal programming (i.e. \( \eta \)) from 1.

In conventional (crisp) goal programming problem GP, the under and/or over deviational variables \( \eta_i \) and \( \rho_i \) \((i=1, 2)\) are included in the achievement function (objective function) for minimizing them but here, as explained, only \( \eta_i \) will be included. Since there are many choices of the objective function for the (crisp) linear goal programming problem we have similar formulations for the fuzzy goal programming problem as well. For example if we take \textit{minsum GP} approach, the fuzzy goal programming formulation corresponding to the given problem (P10a) & (P10b) introducing the negative and positive deviational variables \( \eta_i \) and \( \rho_i \) are given as problem (P11a) & (P11b) respectively for the segment professionals & segment students.
Segment: professionals

Minimize $u$

Subject to

$\mu_{A_1}(x) + \eta - \rho_1 = 1$

$\mu_{Z_{p1}}(x) + \eta_{p1} - \rho_{p1} = 1$

$u \geq w_{p1} \eta_{p1}$

$\eta_{p1} \rho_{p1} = 0$

$\eta_{p1}, \rho_{p1} \geq 0$

$X_1 \in S_1: X_1 \geq 0$ & integers

Segment: students

Minimize $u$

Subject to

$\mu_{A_2}(x) + \eta_2 - \rho_2 = 1$

$\mu_{Z_{p2}}(x) + \eta_{p2} - \rho_{p2} = 1$

$u \geq w_{p2} \eta_{p2}$

$\eta_{p2} \rho_{p2} = 0$

$\eta_{p2}, \rho_{p2} \geq 0$

$X_2 \in S_2: X_2 \geq 0$ & integers

6. Numerical Illustration

Two different print media (newspapers and magazines), television media channels & websites have been chosen for two types of users (professionals & students). Also due weightages are given to different products in each category of users and their respective individual reach & budgets have been shown in Table 1. The total budget allocated initially for professionals segment & students segment was Rs 125,00,000 & Rs 110,00,000 respectively. For different weightages combination the problem for both the segments is solved. The corresponding value of $u$ for different weightages is shown in Table 2. $W_1$, $W_2$ & $W_3$ corresponds to the weightages given to laptops, notebooks & printers/scanners respectively. Problem is solved using optimization software LINGO.

<table>
<thead>
<tr>
<th>Table 1. Advertising reach &amp; budget for different products</th>
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<tbody>
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<td><strong>Professionals</strong></td>
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<td>$W_1=.33, W_2=.33, W_3=.33$</td>
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<tr>
<td>Notebook</td>
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<tr>
<td>Printers</td>
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<tr>
<td>$W_1=.5, W_2=.2, W_3=.1$</td>
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<tr>
<td><strong>Reach</strong></td>
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<tr>
<td>Laptops</td>
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<td>Printers</td>
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<table>
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<th>Table 2. $u$ vs weightages</th>
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<td>$W_1=.33, W_2=.33, W_3=.33$</td>
</tr>
<tr>
<td>$W_1=.5, W_2=.3, W_3=.2$</td>
</tr>
</tbody>
</table>

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It can be seen that by giving different weightages to different products in professional as well as student segment, expected number of customers obtained are different. Such sensitivity analysis usually helps management in deciding the preferences to be given to individual products. For example, suppose management wants to promote its notebook amongst professionals heavily so accordingly it can set high preferences to this product category. This problem on solving will generate the number of different media advertisements that can be allotted amongst different media so as to obtain the expected increase in customers. Also the total budget that will be used can be computed. Hence the model is flexible enough to perform post optimality analysis and therefore helps in deciding strategically according to the needs and aspirations of management.

7. Concluding remarks & scope for future research

Fuzzy systems have gained more and more attention from researchers and practitioners in various fields. In one such field of advertising and media selection, this paper makes use of fuzzy concepts to develop a media selection and allocation model for multiple products that need to accommodate different market segments. The aim is to maximize the expected number of customers for all the products in multiple segments under fuzzy environment. A compromised solution is obtained using the crisp goal programming approach suggested by Mohamed (1991). The model is flexible enough and various post optimality analysis can be performed depending on the needs and requirements of the management. However there are certain areas or dimensions in which the paper can easily be extended. Fuzzy predictions of the triangular fuzzy statistical data have been defuzzified using Heilpern’s defuzzifier and a crisp multi-objective media selection model has been developed using the defuzzified values. Other than Heilpern’s approach, several analytical methods have been proposed such as a recent one by Saneifard (2011) to deal with ranking fuzzy numbers. These approaches not only deal with triangular fuzzy numbers but also with trapezoidal fuzzy numbers. The problem takes into account the fuzzy advertising cost budget as the prime constraint, other than cost budget, a firm can have other constraints such as diversification constraints, advertising breaks constraints or mature product constraints. This can be taken as a scope of future research. The problem has been illustrated for two different segments. However it can also be handled for more than two segments.

References


### Table 4. Expected customer increase rate (%)

| Product class | Professionals | | | | Students | | | | |
|---------------|---------------|---------------|----------------|---------------|----------------|---------------|----------------|---------------|
|               | MG | NP | TV | WS | Total | MG | NP | TV | WS | Total |
| LP            | 4  | 3  | 4  | 4  | 15   | 4  | 3  | 5  | 4  | 15   |
| NB            | 7  | 5  | 2  | 6  | 20   | 8  | 3  | 2  | 4  | 16   |
| PRN           | 10 | 5  | 3  | 2  | 19   | 7  | 5  | 3  | 2  | 20   |

### Table 5. Advertising presence in publications for different segments for Laptops

| Segments | Publications | Professionals | | | Students | | | | |
|----------|--------------|---------------|----------------|---------------|----------------|---------------|----------------|---------------|
|          |              | FR(S₁) | FAUC ('00) | | | FR (S₂) | FAUC ('00) | | |
| MG       | MG1(FP)      | x₁₁₁₁   | (6,12) | 250,270,290 | x₁₂₁₁   | (10,20) | 250,270,310 | | |
|          | MG1(OP)      | x₁₁₁₂   | (18,36) | 200,220,240 | x₁₂₁₂   | (15,30) | 200,220,240 | | |
|          | MG 2(FP)     | x₁₁₂₁   | (6,12) | 320,355,370 | x₁₂₂₁   | (10,20) | 300,340,360 | | |
|          | MG 2(OP)     | x₁₁₂₂   | (12,24) | 300,320,340 | x₁₂₂₂   | (15,30) | 200,220,240 | | |
|          | MG 3(FP)     | x₁₁₃₁   | (6,12) | 340,365,390 | x₁₂₃₁   | (10,12) | 350,370,410 | | |
|          | MG 3(OP)     | x₁₁₃₂   | (10,15) | 300,320,340 | x₁₂₃₂   | (15,15) | 300,320,340 | | |
|          | MG 4(FP)     | x₁₁₄₁   | (6,12) | 330,350,370 | x₁₂₄₁   | (6,12) | 300,330,360 | | |
|          | MG 4(OP)     | x₁₁₄₂   | (10,15) | 300,320,340 | x₁₂₄₂   | (10,15) | 200,220,240 | | |
| NP       | NP 1(FP)     | x₁₁₁₁   | (25,50) | 230,250,270 | x₁₂₁₁   | (30,70) | 230,250,270 | | |
|          | NP 1(OP)     | x₁₁₁₂   | (50,100) | 200,210,220 | x₁₂₁₂   | (50,100) | 200,210,220 | | |
|          | NP 2(FP)     | x₁₁₂₁   | (25,50) | 300,325,350 | x₁₂₂₁   | (25,50) | 250,270,290 | | |
|          | NP 2(OP)     | x₁₁₂₂   | (50,100) | 200,210,220 | x₁₂₂₂   | (50,100) | 200,210,220 | | |
|          | NP 3(FP)     | x₁₁₃₁   | (50,100) | 200,225,250 | x₁₂₃₁   | (30,60) | 200,220,260 | | |
|          | NP 3(OP)     | x₁₁₃₂   | (100,150) | 200,210,220 | x₁₂₃₂   | (50,100) | 200,210,220 | | |
|          | NP 4(FP)     | x₁₁₄₁   | (50,100) | 200,210,220 | x₁₂₄₁   | (50,100) | 200,250,300 | | |
|          | NP 4(OP)     | x₁₁₄₂   | (100,150) | 200,210,220 | x₁₂₄₂   | (50,100) | 200,210,220 | | |
| TV        | CH1(P)       | x₁₁₁₁   | (500,1000) | 570,605,620 | x₁₂₁₁   | (700,1000) | 570,600,650 | | |
|          | CH1(OT)      | x₁₁₁₂   | (1000,2000) | 400,450,500 | x₁₂₁₂   | (700,1500) | 400,450,500 | | |
|          | CH 2(P)      | x₁₁₂₁   | (600,900) | 520,540,560 | x₁₂₂₁   | (600,900) | 550,600,630 | | |
|          | CH 2(OT)     | x₁₁₂₂   | (600,1000) | 400,450,500 | x₁₂₂₂   | (500,1000) | 400,450,500 | | |
| WS        | WS 1(P)      | x₁₁₁₁   | (1000,1500) | 450,505,520 | x₁₂₁₁   | (600,1000) | 350,400,430 | | |
|          | WS 1(OT)     | x₁₁₁₂   | (1500,2000) | 400,425,450 | x₁₂₁₂   | (1000,1500) | 200,210,220 | | |
|          | WS 2(P)      | x₁₁₂₁   | (1000,1500) | 410,445,500 | x₁₂₂₁   | (600,1000) | 400,440,460 | | |
|          | WS 2(OT)     | x₁₁₂₂   | (1200,2500) | 400,415,450 | x₁₂₂₂   | (1000,2500) | 200,210,220 | | |

Appendix

LP: laptop; NB: notebook; PRN: printers & scanners; MG: Magazines; NP: newspapers; WS: websites; FAUC: fuzzy advertising unit cost; FR: frequency range; FP: Front page; OP: other page; PT: Prime time; OT: Other time
Table 6. Advertising Presence in Publications For Different Segments for Notebooks

| Segments | Professionals | | Students | |
|----------|---------------|------------------|-----------|
|          | PB’s          | FR (x)           | FAUC (’00) | FR (x)           | FAUC (’00) |
|          | MG1(FP)       | $x_{2111}$ (6,12)| 250,270,290| $x_{2211}$ (6,12)| 250,270,310|
|          | MG1(OP)       | $x_{2112}$ (18,36)| 200,220,240| $x_{2212}$ (18,36)| 200,220,240|
|          | MG2(FP)       | $x_{2121}$ (6,12)| 320,355,370| $x_{2221}$ (6,12)| 300,340,360|
|          | MG2(OP)       | $x_{2122}$ (12,24)| 300,320,340| $x_{2222}$ (12,24)| 200,220,240|
|          | MG3(FP)       | $x_{2131}$ (6,12)| 340,365,390| $x_{2231}$ (6,12)| 320,370,400|
|          | MG3(OP)       | $x_{2132}$ (10,15)| 300,320,340| $x_{2232}$ (10,15)| 300,320,340|
|          | MG4(FP)       | $x_{2141}$ (6,12)| 330,350,370| $x_{2241}$ (6,12)| 300,330,360|
|          | MG4(OP)       | $x_{2142}$ (10,15)| 300,320,340| $x_{2242}$ (10,15)| 200,220,240|
|          | NP1(FP)       | $x_{2211}$ (25,50)| 230,250,270| $x_{2221}$ (25,50)| 230,250,270|
|          | NP1(OP)       | $x_{2212}$ (50,100)| 200,210,220| $x_{2222}$ (50,100)| 200,210,220|
|          | NP2(FP)       | $x_{2221}$ (25,50)| 300,325,350| $x_{2222}$ (25,50)| 250,270,290|
|          | NP2(OP)       | $x_{2222}$ (50,100)| 200,210,220| $x_{2222}$ (50,100)| 200,210,220|
|          | NP3(FP)       | $x_{2231}$ (50,100)| 200,225,250| $x_{2231}$ (50,100)| 200,220,260|
|          | NP3(OP)       | $x_{2232}$ (100,150)| 200,210,220| $x_{2232}$ (100,150)| 180,210,220|
|          | NP4(FP)       | $x_{2241}$ (50,100)| 200,210,220| $x_{2241}$ (50,100)| 200,250,300|
|          | NP4(OP)       | $x_{2242}$ (100,150)| 200,210,220| $x_{2242}$ (100,150)| 200,210,220|
|          | TV CH1(PT)    | $x_{2311}$ (500,1000)| 570,605,620| $x_{2321}$ (500,1000)| 570,600,650|
|          | CH1(OT)       | $x_{2312}$ (1000,2000)| 400,450,500| $x_{2321}$ (1000,2000)| 400,450,500|
|          | CH2(PT)       | $x_{2321}$ (600,900)| 520,540,560| $x_{2322}$ (600,900)| 550,600,630|
|          | CH2(OT)       | $x_{2322}$ (600,1000)| 400,450,500| $x_{2322}$ (600,1000)| 400,450,500|
|          | WS1(PT)       | $x_{2411}$ (1000,1500)| 450,505,520| $x_{2421}$ (1000,1500)| 350,400,430|
|          | WS1(OT)       | $x_{2412}$ (1500,2000)| 400,425,450| $x_{2421}$ (1500,2000)| 200,210,220|
|          | WS2(PT)       | $x_{2421}$ (1000,1500)| 410,445,500| $x_{2421}$ (1000,1500)| 400,440,460|
|          | WS2(OT)       | $x_{2422}$ (1200,2500)| 400,415,450| $x_{2422}$ (1200,3000)| 200,210,220|

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Table 7. Advertising Presence in Publications for different segments for printers/scanners

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<th>Segments</th>
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