A fuzzy analytic hierarchy process tool to evaluate computer-aided manufacturing software alternatives

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Abstract

In this paper, an intelligent approach is presented to help companies select most suitable computer-aided manufacturing (CAM) software among a set of existing alternatives for their current and future needs. For this purpose, a fuzzy analytic hierarchy process (AHP) method integrating the fuzzy logic and AHP methods is used to carry out CAM selection process more effectively, easily and applicable for a company. Shortly, the objectives of the research are; to define a step-by-step approach for an effective CAM software selection problem. The applicability of the proposed approach is also illustrated on a case study.

Keywords: Fuzzy logic, multiple-criteria decision making, analytic hierarchy process, computer-aided manufacturing.

1. Introduction

The winds of globalization have affected most manufacturing companies especially in the developing countries and put them under intense competitive pressures. Major changes are being experienced with respect to resources, markets, manufacturing processes and product strategies. Many of the efforts in this direction are being carried forth under the banner of computer integrated manufacturing (CIM) system. A CIM includes all of the engineering functions of CAD/CAM, but it also includes the firm’s business functions that are related to manufacturing. The ideal CIM system applies computer and communications technology to all of the operational functions and information processing functions in manufacturing from order receipt, through design and production, to product shipment. A CIM system is capital intensive due to hardware and software requirements. As a result, it is essential that it achieves high levels of flexibility and productivity compared to traditional manufacturing systems (Groover, 2001).

Computer-aided design and manufacturing (CAD/CAM) systems, as integrated parts of a CIM system, realizes all kinds of design and manufacturing activities in a product development environment (PDE) by using computer technology in a good manner. The
The implementation process of a CAD/CAM system can be difficult due to the fact that the amount of investment required is generally high relating to the degree of integration. That is why designing, planning and realizing of these systems have received substantial attention in recent years due to high initial investment cost of such systems as well as unprecedented mixture of success and horror stories on their implementations.

One of the most important issues in a CAD/CAM implementation is to select the most satisfying software packages based upon the needs and expectations of a company. In the presence of multiple criteria and alternatives for proper software selection, the multiple-criteria decision making (MCDM) problem arises. The analytic hierarchy process (AHP), as one of the most commonly used MCDM methods in literature, is selected. Next more detail of the AHP is presented.

The conventional AHP has been used for MCDM problems for years. But, in the conventional AHP, the pair wise comparisons for each level with respect to the goal of the best alternative selection are conducted using a nine-point scale. That is why the application of Saaty’s AHP has some shortcomings as follows (Saaty, 1981); (1) the AHP method is mainly used in nearly crisp decision applications, (2) the AHP method creates and deals with a very unbalanced scale of judgment, (3) the AHP method does not take into account the uncertainty associated with the mapping of one’s judgment to a number, (4) ranking of the AHP method is rather imprecise, (5) the subjective judgment, selection and preference of decision-makers have great influence on the AHP results.

In addition, a decision maker’s requirements on evaluating process always contain ambiguity and multiplicity of meaning. Furthermore, it is also recognized that human assessment on qualitative attributes is always subjective and thus imprecise. Therefore, conventional AHP seems inadequate to capture decision maker’s requirements explicitly. In order to model this kind of uncertainty in human preference, fuzzy sets could be incorporated with the pair wise comparison as an extension of AHP.

The fuzzy AHP approach allows a more accurate description of the decision making process. The earliest work in fuzzy AHP appeared in Van Laarhoven and Pedrycz (1983), which compared fuzzy ratios described by triangular membership functions. Logarithmic least square was used to derive the local fuzzy priorities. Later, using geometric mean, Buckley (1985) determined fuzzy priorities of comparison, whose membership functions were trapezoidal. By modifying the Van Laarhoven and Pedrycz method, Boender et al. (1989) presented a more robust approach to the normalization of the local priorities. Deng (1999) proposed a simple, improved, and sophisticated approach using fuzzy logic.

The main objective of this paper is to define a step-by-step approach for an effective CAM software selection. In final section, the proposed approach was applied to a company as a case study in order to prove its applicability on a real-life system, which is the leading company with ISO 9001 certification in designing and manufacturing all kinds of cutting tools.
The rest of this paper is organized as follows. In the next section, the related literature on the subject is given, and then the proposed approach is presented, as in last section, a case study is given to show the applicability of the proposed approach.

2. Related research

Fuzzy set theory is a mathematical theory designed to model the vagueness or imprecision of human cognitive processes that pioneered by Zadeh (Lootsma, 1997). This theory is basically a theory of classes with unsharp boundaries. What is important to recognize is that any crisp theory can be fuzzified by generalizing the concept of a set within that theory to the concept of a fuzzy set. The stimulus for the transition from a crisp theory to a fuzzy one derives from the fact both the generality of a theory and its applicability to real world problems are enhanced by replacing the concept of a crisp set with a fuzzy set (Zadeh, 1994). The key idea of fuzzy set theory is that an element has a degree of membership in a fuzzy set (Negoita, 1985 and Zimmermann, 1985). The membership function represents the grade of membership of an element in a set. The membership values of an element vary between 0 and 1. Elements can belong to a set in a certain degree and elements can also belong to multiple set. Fuzzy set allows the partial membership of elements. Transition between membership and non-membership is gradually. Membership function maps the variation of value of linguistic variables into different linguistic classes. The adaptation of membership function for a given linguistic variable under a given situation is done in three ways; a) experts previous knowledge about the linguistic variable; b) using simple geometric forms having slopes (triangular, trapezoidal or s-functions ) as per the nature of the variable; and c) by trial and error learning process.

The concept of fuzzy extent analysis is applied to solve the fuzzy reciprocal matrix for determining the criteria importance and alternative performance. To avoid the complex and unreliable process of comparing fuzzy utilities, the alpha-cut concept is used to transform the fuzzy performance matrix representing the overall performance of all alternatives with respect to each criterion into an interval performance matrix. Incorporated with the decision maker’s attitude towards risk, an overall performance index is obtained for each alternative across all criteria by applying the concept of the degree of similitaty to the ideal solution using the vector matching function. Because of the accuracy of fuzzy AHP method in a decision making process, it has been applied to many different areas such as; Kuo et al. (2002) developed a decision support system using the fuzzy AHP to locate new convenience store. Murtaza (2003) presented a fuzzy version of AHP to country risk assessment problem. Enea and Piazza (2004) used a fuzzy extension of the AHP for project selection and focused on the constraints that have to be considered within fuzzy AHP in order to take in account all the available information. Weck et al. (1997) evaluated alternative production cycles using the extended fuzzy AHP method. Lee et al. (2001) proposed a fuzzy AHP approach in modular product design complemented with a case example to validate its feasibility in a real company. Ayag (2005a, 2005b) also presented an integrated approach to evaluating conceptual design alternatives in a new product development (NPD) environment using AHP-simulation and fuzzy AHP-simulation. Piippo et al. (1999) used group decision support system (GDSS) for a real-life CAD-system selection application for an industrial company. Ayag (2002) developed an AHP-based
simulation model for implementation and analysis of computer-aided systems (CAx). Cheng and Mon (1994) evaluated weapon system by AHP based on fuzzy scales.

3. Proposed approach

In this paper, an intelligent approach, where fuzzy AHP is used together for CAM software selection is proposed. As shown in figure 1, this proposed approach is summarized step-by-step. In practice, the fuzzy AHP method has quite time-consuming implementation steps, especially if they are carried out manually. For instance, as the numbers of criteria and alternatives increase, the dimension of problem naturally expands such as an evaluation matrix with great deal of the columns and lines. This means too long and boring calculation process. Therefore, in this study, an Excel template was designed to facilitate these efforts required for both techniques.

To use this approach, first the criteria and alternatives for each system should be well-defined. These criteria are used to evaluate the number of software alternatives for each system in market as a critical part of CAM software selection. These criteria that may change from a company to another should be well-defined by the project team according to the needs and the goals of company. Alternatives can be obtained from both their vendors and other sources in the market. Second, the fuzzy AHP is introduced (i.e. fuzzy pair wise comparison and the steps of the process) as follows;

![Diagram](image)

**Figure 1.** An integrated approach to CAM software selection problem
Fuzzy representation of pair wise comparison; a hierarchy of software selection problem of a CAM system needs to be established before performing the pair wise comparison of AHP. After constructing a hierarchy for the problem, the decision maker is asked to compare the elements at a given level on a pair wise basis to estimate their relative importance in relation to the element at the immediate proceeding level. In conventional AHP, the pair wise comparison is made by using a ratio scale. A frequently used scale is the nine-point scale (Saaty 1989, Table 1) which shows the participants’ judgments or preferences among the options such as equally important, weakly more important, strongly more important, very strongly more important, and absolutely more important preferred. Even though the discrete scale of 1-9 has the advantages of simplicity and easiness for use, it does not take into account the uncertainty associated with the mapping of one’s perception or judgment to a number.

In this study, triangular fuzzy numbers, 1 to 9, are used to represent subjective pair wise comparisons of selection process in order to capture the vagueness. A fuzzy number is a special fuzzy set $F = \{ (x, \mu_F(x)) | x \in \mathbb{R} \}$, where $x$ takes values on the real line, $\mathbb{R} : -\infty < x < +\infty$ and $\mu_F(x)$ is a continuous mapping from $\mathbb{R}$ to the closed interval $[0, 1]$. A triangular fuzzy number denoted as $M = (l, m, u)$, where $l \leq m \leq u$, has the following triangular type membership function;

Table 1. Definition and membership function of fuzzy number (Ayag, 2005b)

<table>
<thead>
<tr>
<th>Intensity of Importance $^1$</th>
<th>Fuzzy number</th>
<th>Definition</th>
<th>Membership</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>Equally important/preferred</td>
<td>(1, 1, 2)</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>Moderately more important/preferred</td>
<td>(2, 3, 4)</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>Strongly more important/preferred</td>
<td>(4, 5, 6)</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>Very strongly more important/preferred</td>
<td>(6, 7, 8)</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td>Extremely more important/preferred</td>
<td>(8, 9, 10)</td>
</tr>
</tbody>
</table>

$^1$Fundamental scale used in pair wise comparison (Saaty, 1989)

$$
\mu_F(x) = \begin{cases}
0 & x < l \\
\frac{x - l}{m - l} & l \leq x \leq m \\
\frac{u - x}{u - m} & m \leq x \leq u \\
0 & x > u
\end{cases}
$$
Alternatively, by defining the interval of confidence level \( \alpha \), the triangular fuzzy number can be characterized as:

\[
\forall \alpha \in [0,1] \quad \tilde{M}_\alpha = [l^\alpha, u^\alpha] = [(m-l)\alpha+l, (u-m)\alpha+u]
\]

Some main operations for positive fuzzy numbers are described by the interval of confidence, by Kaufmann and Gupta (1985) as given below;

\[
\forall m_L, m_R, n_L, n_R \in R^+, \tilde{M}_\alpha = [m_L^\alpha, m_R^\alpha], \tilde{N}_\alpha = [n_L^\alpha, n_R^\alpha] \alpha \in [0,1]
\]

\[
\tilde{M} \oplus \tilde{N} = [m_L^\alpha + n_L^\alpha, m_R^\alpha + n_R^\alpha], \quad \tilde{M} \odot \tilde{N} = [m_L^\alpha - n_L^\alpha, m_R^\alpha - n_R^\alpha]
\]

The triangular fuzzy numbers (TFNs), \( 1 \) to \( 9 \), are utilized to improve the conventional nine-point scaling scheme. In order to take the imprecision of human qualitative assessments into consideration, the five TFNs are defined with the corresponding membership function as shown in figure 2.

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**Figure 2.** Fuzzy membership function for linguistic values for criteria or alternatives

Steps of fuzzy AHP approach: The AHP method is also known as an eigenvector method. It indicates that the eigenvector corresponding to the largest eigenvalue of the pair wise comparisons matrix provides the relative priorities of the factors, and preserves ordinal preferences among the alternatives. This means that if an alternative is preferred to another, its eigenvector component is larger than that of the other. A vector of weights obtained from the pair wise comparisons matrix reflects the relative performance of the various factors. In the fuzzy AHP, triangular fuzzy numbers are utilized to improve the scaling scheme in the judgment matrices, and interval arithmetic is used to solve the fuzzy eigenvector (Cheng and Mon, 1994).
In this study, the five-step-procedure is defined for fuzzy AHP as follows;

**Step 1. Comparing the performance score:** Triangular fuzzy numbers \((1, 3, 5, 7, 9)\) are used to indicate the relative strength of each pair of elements in the same hierarchy.

**Step 2. Constructing the fuzzy comparison matrix:** By using triangular fuzzy numbers, via pair wise comparison, the fuzzy judgment matrix \(\tilde{A} \left( a_{ij} \right)\) is constructed as given below;

\[
\tilde{A} = \begin{bmatrix}
1 & a_{12} & \ldots & a_{1n} \\
a_{21} & 1 & \ldots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{n1} & a_{n2} & \ldots & 1
\end{bmatrix}
\]

where, \(a_{ij}^{a} = 1\), if \(i\) is equal \(j\), and \(a_{ij}^{a} = 1, 3, 5, 7, 9\) or \(1^{-1}, 3^{-1}, 5^{-1}, 7^{-1}, 9^{-1}\), if \(i\) is not equal \(j\).

**Step 3. Solving fuzzy eigenvalues:** A fuzzy eigenvalue, \(\tilde{\lambda}\), is a fuzzy number solution to

\[
\tilde{A} \tilde{x} = \tilde{\lambda} \tilde{x}
\]

where \(\tilde{A}\) is a \(n \times n\) fuzzy matrix containing fuzzy numbers \(a_{ij}\) and \(\tilde{x}\) is a non-zero \(nx1\) fuzzy vector containing fuzzy numbers \(\tilde{x}_i\). To perform fuzzy multiplications and additions using the interval arithmetic and \(\alpha-cut\), the equation \(\tilde{A} \tilde{x} = \tilde{\lambda} \tilde{x}\) is equivalent to

\[
\tilde{A} = \begin{bmatrix}
\tilde{a}_{ij} \\
\tilde{x}_i
\end{bmatrix}, \tilde{x}' = \begin{bmatrix}
\tilde{x}_1, \ldots, \tilde{x}_n
\end{bmatrix}, \tilde{a}_{ij} = \begin{bmatrix}
a_{ij}^{a}, a_{ij}^{\alpha}
\end{bmatrix}, \tilde{x}_i = \begin{bmatrix}
a_{ii}^{a}, a_{ii}^{\alpha}
\end{bmatrix}, \tilde{\lambda} = \begin{bmatrix}
\lambda_{ij}^{a}, \lambda_{ij}^{\alpha}
\end{bmatrix}
\]

for \(0 < \alpha \leq 1\) and all \(i, j\), where \(i=1,2,\ldots,n, j=1,2,\ldots,n\)

\(\alpha-cut\) is known to incorporate the experts or decision maker(s) confidence over his/her preference or the judgments. Degree of satisfaction for the judgment matrix \(\tilde{A}\) is estimated by the index of optimism \(\mu\). The larger value of index \(\mu\) indicates the higher degree of optimism. The index of optimism is a linear convex combination (Lee, 1999) defined as;

\[
\tilde{a}_{ij}^{a} = \mu a_{ij}^{a} + (1 - \mu)a_{ij}^{\alpha}, \quad \forall \mu \in [0,1]
\]
While $\alpha$ is fixed, the following matrix can be obtained after setting the index of optimism, $\mu$, in order to estimate the degree of satisfaction.

$$\tilde{A} = \begin{bmatrix}
1 & a_{12} & \ldots & a_{1n} \\
-\alpha & 1 & \ldots & a_{2n} \\
\ldots & \ldots & \ldots & \ldots \\
a_{n1} & a_{n2} & \ldots & 1
\end{bmatrix}$$

**Step 4. Normalization of the matrices:** Normalization of both the matrix of paired comparisons and calculation of priority weights (approx. criteria weights), and the matrices and priority weights for alternatives with respect to each criterion are also done before calculating $\lambda_{\text{max}}$. In order to control the result of the method, the consistency ratio for each of the matrices and overall inconsistency for the hierarchy calculated. The deviations from consistency are expressed by the following equation consistency index and the measure of inconsistency is called the consistency index (CI);

$$CI = \frac{\lambda_{\text{max}} - n}{n - 1}$$  \hspace{1cm} (4)

The consistency ratio (CR) is used to estimate directly the consistency of pairwise comparisons. The CR is computed by dividing the CI by a value obtained from a table of Random Consistency Index (RI);

$$CR = \frac{CI}{RI}$$  \hspace{1cm} (5)

If the CR less than 10%, the comparisons are acceptable, otherwise they should be repeated until reached to the CR, less than 10%. RI is the average index for randomly generated weights (Saaty, 1981).

**Step 5. Calculation of priority weights for each alternative:** The priority weight of each alternative can be obtained by multiplying the matrix of evaluation ratings by the vector of criterion weights and summing over all criteria. Expressed in conventional mathematical notation;

$$\text{Weighted evaluation for alternative } k = \sum_{i=1}^{t} (\text{criterionweight}_i \times \text{evaluationrating}_{ik})$$  \hspace{1cm} (6)

for $i=1,2,...,t$ ( $t$ : total number of criteria )

After calculating the weight for each alternative, the overall consistency index is also calculated that it should be less than 10% for consistency on all judgments.
4. Case Study

Above, an intelligent approach has been proposed for a CAM software selection. In this section, a case study is presented to prove its applicability and validity. Therefore, a cutting tool manufacturer, a leading company in designing and manufacturing of all kinds of cutting tools was chosen. Firstly, the 7 criteria and 5 alternatives (A1, A2, A3, A4, and A5) were defined for CAM software selection as shown in Table 2.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>System (hardware and software) cost</td>
<td>C1</td>
</tr>
<tr>
<td>Installation and training cost</td>
<td>C2</td>
</tr>
<tr>
<td>Annual maintenance cost</td>
<td>C3</td>
</tr>
<tr>
<td>On-line support and service on-time</td>
<td>C4</td>
</tr>
<tr>
<td>Compatibility to existing software and hardware</td>
<td>C5</td>
</tr>
<tr>
<td>Ease of learning and use</td>
<td>C6</td>
</tr>
<tr>
<td>Simplicity of tool path creation and accuracy of post processing</td>
<td>C7</td>
</tr>
</tbody>
</table>

Next the fuzzy AHP was carried out to evaluate the CAD software alternatives and ranked them by weight. Fuzzy triangular numbers \((1, 3, 5, 7, 9)\) in Table 1 was used to rate both all the alternatives with respect to one criterion at a time, and the criteria on each other. Both the pair wise comparison matrix of criteria for each level, and the pair wise comparison matrix of alternatives with respect to the first criterion, system cost are given in Table 3 and 4.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>C2</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>C3</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>C4</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>C5</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>C6</td>
<td>5</td>
<td>7</td>
<td>5</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>C7</td>
<td>9</td>
<td>7</td>
<td>5</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 4. The fuzzy comparison matrix for the alternatives with respect to the first criterion – system cost (C1) using TFNs

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
<th>A5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>A2</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>A3</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>A4</td>
<td>5</td>
<td>3</td>
<td>5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>A5</td>
<td>7</td>
<td>3</td>
<td>9</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

The lower limit and upper limit of the fuzzy numbers with respect to the \( \alpha \) were defined as follows by applying Eq. 2, and all calculations are shown in Table 5 and 6:

\[
\tilde{1}_\alpha = [1, 3 - 2\alpha], \quad 3_\alpha = [1 + 2\alpha, 5 - 2\alpha], \quad \tilde{3}_\alpha^{-1} = \left[ \frac{1}{5 - 2\alpha}, \frac{1}{1 + 2\alpha} \right].
\]

\[
\tilde{5}_\alpha = [3 + 2\alpha, 7 - 2\alpha], \quad \tilde{5}_\alpha^{-1} = \left[ \frac{1}{7 - 2\alpha}, \frac{1}{3 + 2\alpha} \right].
\]

\[
\tilde{7}_\alpha = [5 + 2\alpha, 9 - 2\alpha], \quad \tilde{7}_\alpha^{-1} = \left[ \frac{1}{9 - 2\alpha}, \frac{1}{5 + 2\alpha} \right].
\]

\[
\tilde{9}_\alpha = [7 + 2\alpha, 11 - 2\alpha], \quad \tilde{9}_\alpha^{-1} = \left[ \frac{1}{11 - 2\alpha}, \frac{1}{7 + 2\alpha} \right].
\]

Table 5. The \( \alpha \)-cuts fuzzy comparison matrix for the criteria (\( \alpha = 0.5 \))

<table>
<thead>
<tr>
<th>Criteria</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>1</td>
<td>[1, 2]</td>
<td>[1, 2]</td>
<td>[2, 4]</td>
<td>[2, 4]</td>
<td>[4, 6]</td>
<td>[8, 10]</td>
</tr>
<tr>
<td>C2</td>
<td>[1/2, 1]</td>
<td>1</td>
<td>[1, 2]</td>
<td>[2, 4]</td>
<td>[1, 2]</td>
<td>[2, 4]</td>
<td>[2, 4]</td>
</tr>
<tr>
<td>C3</td>
<td>[1/2, 1]</td>
<td>[1/2, 1]</td>
<td>1</td>
<td>[2, 4]</td>
<td>[2, 4]</td>
<td>[2, 4]</td>
<td>[6, 8]</td>
</tr>
<tr>
<td>C4</td>
<td>[1/4, 1/2]</td>
<td>[1/4, 1/2]</td>
<td>[1/2, 1]</td>
<td>1</td>
<td>[2, 4]</td>
<td>[1, 2]</td>
<td>[4, 6]</td>
</tr>
<tr>
<td>C5</td>
<td>[1/4, 1/2]</td>
<td>[1/2, 1]</td>
<td>[1/4, 1/2]</td>
<td>[1/4, 1/2]</td>
<td>1</td>
<td>[2, 4]</td>
<td>[4, 6]</td>
</tr>
<tr>
<td>C6</td>
<td>[1/6, 1/4]</td>
<td>[1/4, 1/2]</td>
<td>[1/4, 1/2]</td>
<td>[1/2, 1]</td>
<td>[1/4, 1/2]</td>
<td>1</td>
<td>[1, 2]</td>
</tr>
<tr>
<td>C7</td>
<td>[1/10, 1/6]</td>
<td>[1/4, 1/2]</td>
<td>[1/8, 1/6]</td>
<td>[1/6, 1/4]</td>
<td>[1/6, 1/4]</td>
<td>[1/2, 1]</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 5. The $\alpha$–cuts fuzzy comparison matrix for the alternatives with respect to the first criterion – system cost (C1) ($\alpha = 0.5$)

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>A</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
<th>A5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>1</td>
<td>[2, 4]</td>
<td>[2, 4]</td>
<td>[4, 6]</td>
<td>[6, 8]</td>
</tr>
<tr>
<td>A2</td>
<td>[1/4, 1/2]</td>
<td>1</td>
<td>[1, 2]</td>
<td>[2, 4]</td>
<td>[2, 4]</td>
</tr>
<tr>
<td>A3</td>
<td>[1/4, 1/2]</td>
<td>[1/2, 1]</td>
<td>1</td>
<td>[4, 6]</td>
<td>[8, 10]</td>
</tr>
<tr>
<td>A4</td>
<td>[1/6, 1/4]</td>
<td>[1/4, 1/2]</td>
<td>[1/6, 1/4]</td>
<td>1</td>
<td>[1, 2]</td>
</tr>
<tr>
<td>A5</td>
<td>[1/8, 1/6]</td>
<td>[1/4, 1/2]</td>
<td>[1/10, 1/8]</td>
<td>[1/2, 1]</td>
<td>1</td>
</tr>
</tbody>
</table>

The values, $\alpha = 0.5$ and $\mu = 0.5$ were substituted into the above expression into fuzzy comparison matrices, all the $\alpha$–cuts fuzzy comparison matrices were obtained as follows; Eq. (3) was used to calculate eigenvectors for all comparison matrices. All the necessary calculations (synthesizing the pair wise comparison matrix, calculating the priority vector of a criterion, the CR, $\lambda_{\text{max}}$ using Eq.(1) and the CI, selecting appropriate value of the RC and checking the consistency of the pair wise comparison matrix to check out whether the comparisons of the decision-maker were consistent or not) were carried out. Synthesizing the pair wise comparison matrix of the criteria is performed by dividing each element of the matrix by its column total. The priority vector of the criteria can be obtained by finding the row averages (Table 7).

Table 7. The eigenvector for comparison matrix of the criteria

<table>
<thead>
<tr>
<th>Criteria</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>e-Vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>1,000</td>
<td>1,500</td>
<td>1,500</td>
<td>3,000</td>
<td>3,000</td>
<td>5,000</td>
<td>9,000</td>
<td>0.281</td>
</tr>
<tr>
<td>C2</td>
<td>0.750</td>
<td>1,000</td>
<td>1,500</td>
<td>3,000</td>
<td>3,000</td>
<td>3,000</td>
<td>3,000</td>
<td>0.196</td>
</tr>
<tr>
<td>C3</td>
<td>0.750</td>
<td>0.750</td>
<td>1,000</td>
<td>1,500</td>
<td>3,000</td>
<td>3,000</td>
<td>7,000</td>
<td>0.190</td>
</tr>
<tr>
<td>C4</td>
<td>0.375</td>
<td>0.375</td>
<td>0.750</td>
<td>1,000</td>
<td>3,000</td>
<td>1,500</td>
<td>5,000</td>
<td>0.130</td>
</tr>
<tr>
<td>C5</td>
<td>0.375</td>
<td>0.750</td>
<td>0.375</td>
<td>0.375</td>
<td>1,000</td>
<td>3,000</td>
<td>5,000</td>
<td>0.110</td>
</tr>
<tr>
<td>C6</td>
<td>0.208</td>
<td>0.375</td>
<td>0.375</td>
<td>0.750</td>
<td>0.375</td>
<td>1,000</td>
<td>1,500</td>
<td>0.059</td>
</tr>
<tr>
<td>C7</td>
<td>0.113</td>
<td>0.375</td>
<td>0.146</td>
<td>0.208</td>
<td>0.208</td>
<td>0.750</td>
<td>1,000</td>
<td>0.035</td>
</tr>
</tbody>
</table>

$\lambda_{\text{max}}$ = 7.751
CI = 0.125
RI = 1.32
CR = 0.095 <

Now, the consistency ratio for the matrix of pair wise comparisons of the alternatives with respect to the criterion – system cost (C1) was calculated by using Eq. (3)-(4) (Table 8). The calculations for the matrices of pair wise comparisons of the alternatives for the 6 remaining criteria were done respectively by using the same way, and their consistency ratios were found out that they were less than 0.10. Based on the calculations above, it is said that the consistencies of the judgments in all comparison matrices were also acceptable.
Table 8. The eigenvector for comparison matrix of the alternatives with respect to the first criterion – system cost (C1)

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
<th>A5</th>
<th>e-Vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>1,000</td>
<td>3,000</td>
<td>3,000</td>
<td>5,000</td>
<td>7,000</td>
<td>0,440</td>
</tr>
<tr>
<td>A2</td>
<td>0,375</td>
<td>1,000</td>
<td>1,500</td>
<td>3,000</td>
<td>3,000</td>
<td>0,192</td>
</tr>
<tr>
<td>A3</td>
<td>0,375</td>
<td>0,750</td>
<td>1,000</td>
<td>5,000</td>
<td>9,000</td>
<td>0,249</td>
</tr>
<tr>
<td>A4</td>
<td>0,208</td>
<td>0,375</td>
<td>0,208</td>
<td>1,000</td>
<td>1,500</td>
<td>0,068</td>
</tr>
<tr>
<td>A5</td>
<td>0,146</td>
<td>0,375</td>
<td>0,113</td>
<td>0,750</td>
<td>1,000</td>
<td>0,051</td>
</tr>
</tbody>
</table>

\[
\lambda_{max} = 5.443 \\
CI = 0.111 \\
RI = 1.12 \\
CR = 0.099 < 0.100
\]

Finally, the overall priority weight for each alternative was calculated by using Eq. (6). The overall consistency index was also calculated as 0.091. Because it is smaller than 0.10, all of the judgments are consistent. All results are presented in Table 9.

Table 9. The final ranking of CAM software alternatives

<table>
<thead>
<tr>
<th>Alternative</th>
<th>C1 (0.281)</th>
<th>C2 (0.196)</th>
<th>C3 (0.190)</th>
<th>C4 (0.130)</th>
<th>C5 (0.110)</th>
<th>C6 (0.059)</th>
<th>C7 (0.035)</th>
<th>Overall e-Vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.440</td>
<td>0.394</td>
<td>0.459</td>
<td>0.498</td>
<td>0.505</td>
<td>0.424</td>
<td>0.487</td>
<td>0.450*</td>
</tr>
<tr>
<td>A2</td>
<td>0.192</td>
<td>0.300</td>
<td>0.203</td>
<td>0.182</td>
<td>0.185</td>
<td>0.247</td>
<td>0.203</td>
<td>0.217</td>
</tr>
<tr>
<td>A3</td>
<td>0.249</td>
<td>0.176</td>
<td>0.193</td>
<td>0.173</td>
<td>0.134</td>
<td>0.143</td>
<td>0.159</td>
<td>0.192</td>
</tr>
<tr>
<td>A4</td>
<td>0.068</td>
<td>0.069</td>
<td>0.090</td>
<td>0.090</td>
<td>0.110</td>
<td>0.114</td>
<td>0.079</td>
<td>0.083</td>
</tr>
<tr>
<td>A5</td>
<td>0.051</td>
<td>0.061</td>
<td>0.055</td>
<td>0.057</td>
<td>0.066</td>
<td>0.072</td>
<td>0.072</td>
<td>0.058</td>
</tr>
<tr>
<td>CR</td>
<td>0.099</td>
<td>0.087</td>
<td>0.088</td>
<td>0.084</td>
<td>0.097</td>
<td>0.082</td>
<td>0.072</td>
<td></td>
</tr>
</tbody>
</table>

As seen in table; the alternative with highest weight (0.450) is A1.

5. Conclusions

In this paper, an intelligent approach to a CAM software selection has been presented, which includes using the fuzzy AHP to evaluate a set of software alternatives for CAM. In the approach, triangular fuzzy numbers were introduced into the conventional AHP in order to improve the degree of judgments of decision maker(s). Furthermore, using of fuzzy AHP approach to evaluating alternatives for CAM systems in the following two major advantages; (1) Fuzzy numbers are preferable to extend the range of a crisp comparison matrix of the conventional AHP method, as human judgment in the comparisons of selection criteria and alternatives is really fuzzy in nature, (2) Adoption of fuzzy numbers can allow decision maker(s) to have freedom of estimation regarding the software selection problem of CAM. In addition, the proposed integrated approach
has also no restriction. But, as the number of criteria and alternatives increases, the dimension of the problem gets more complex. In future research, fuzzy ANP method can be used to make the results more reliable.

References


Ayag, Z., A fuzzy AHP-based simulation approach to concept evaluation in a NPD environment. IIE Transactions, 37, 827-842, 2005b.


